



Technical Note

Surface wetting effects on the spreading of liquid droplets impacting a solid surface at low Weber numbers

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Abstract

A modification to the Kurabayashi–Yang equation, for predicting the maximum spreading ratio of a liquid droplet impacting a solid surface, has been made to account for effects of the contact angle between the spreading liquid and the impact surface. A computational fluid dynamics model was used to generate the correction factor, and comparison of the corrected model to experimental data from the literature shows that predictions improved significantly. The average error between the model's predictions and the experimental values dropped from $12.2 \pm 24.8\%$ for the original equation to $3.6 \pm 12.2\%$ for the corrected equation. © 2000 Elsevier Science Ltd. All rights reserved.

1. Introduction

Accurate predictions of the spreading characteristics of a liquid droplet impacting a solid surface are necessary when estimating heat transfer between the solid and the liquid film formed by that droplet. The principal dimension of interest is the diameter of the film which is often expressed in terms of the non-dimensional spreading ratio, β . Of particular interest is the maximum spreading ratio, β_{\max} .

In a previous work, Healy et al. [1] compared the predictions of β from several analytical models to experimental data and found the Kurabayashi–Yang (K–Y) equation [2] to provide the most accurate predic-

tions of β_{\max} . The expression developed by Kurabayashi and later modified by Yang is given by Eq. (1),

$$\frac{We}{2} = \frac{3}{2} \beta_{\max}^2 \left[1 + \frac{3We}{Re} \left(\beta_{\max}^2 \ln \beta_{\max} - \frac{\beta_{\max}^2 - 1}{2} \right) \times \left(\frac{\mu_{\text{drop}}}{\mu_{\text{wall}}} \right)^{0.14} \right] - 6. \quad (1)$$

The average error between the predicted values from this equation and the experimental data was 9.9%, while the standard deviation of these percent errors was 10.3%. Although these errors appear to be relatively small, the corresponding errors in the calculated area covered by the spreading liquid film would be much larger since the area is proportional to β^2 . Clearly, the coverage area of the liquid film directly impacts heat transfer predictions; hence, slight errors in predictions of the spreading ratio can lead to significant errors in heat transfer calculations.

An obvious drawback of the K–Y model is that the wetting effect, i.e. the contact angle between the

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Nomenclature

d	diameter of liquid film during spreading
D	diameter of liquid droplet before impact
Re	impact Reynolds number of droplet = $\rho V D / \mu$
We	impact Weber number of droplet = $\rho V^2 D / \sigma$
V	droplet impact velocity

Greek symbols

β	spreading ratio, d/D
μ	liquid viscosity
θ	contact angle

ρ	density
σ	surface tension

Subscripts

drop	evaluated at initial droplet temperature
KY	predicted by the Kurabayashi–Yang equation
KY, corr	corrected prediction from Kurabayashi–Yang equation
max	maximum
wall	evaluated at wall temperature

spreading liquid film and the solid surface, is not considered in the equation. In the present work, a correction to Eq. (1) is proposed to account for such wetting effects. This study focuses on a range of $We < 150$ to ensure that the droplet does not fragment upon impact. This range is the domain of interest for spray cooling heat transfer. While some of the other models studied in [1] did include the contact angle as a parameter, they have not been further studied here because of their poor performance, vis-à-vis the K–Y model, in matching experimental data.

2. Analysis

Detailed simulation of the droplet impact process was recently performed using computational fluid dynamics [3]. This numerical model simulated the deformation of a spherical droplet impacting normally upon a surface using the level set method developed by Sussman et al. [4]. The model accounts for full shape changes of the liquid as opposed to the simplified geometry (disk-shaped) used to develop Eq. (1). The complete Navier–Stokes equations are solved using finite-difference methods, and the level set technique tracks the motion of the free-surface. The most major modification made to the formulation as it pertains to this investigation was the addition of a specified contact angle. The numerical model predictions were compared to experimental data, with excellent agreement. Results of those simulations showed that the spreading process is highly dependent on We , Re , and the contact angle, θ . The Weber number accounts for the effect of surface tension, Re accounts for viscous effects, and θ provides a means of estimating the wetting behavior of the liquid.

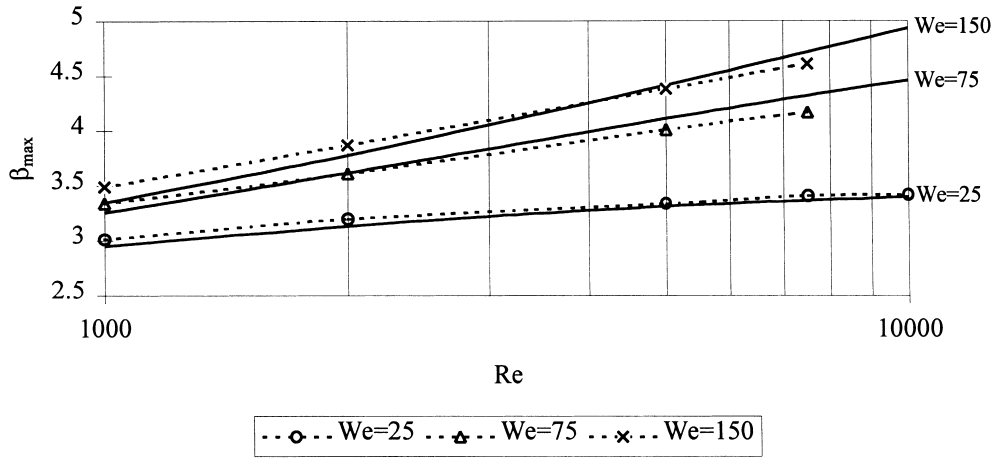
Values of θ near 90° indicate that the liquid is non-wetting, while small values of θ denote a wetting liquid. An examination of Eq. (1) shows that the K–Y equation considers the viscous and surface tension effects, but it omits any dependence on the contact angle. It was, therefore, speculated that the absence of the contact angle in the K–Y equation could be a source of some of its inaccuracies. When the numerical model was compared to predictions from the K–Y equation, it was found that the K–Y equation accurately matched the maximum spreading ratio when the contact angle used in the numerical simulations was near 45° , but larger differences appeared when the spreading liquid had a significantly different contact angle. An example of these findings is displayed in Fig. 1, where values of β_{\max} predicted by the detailed CFD model [3,4], and those calculated by the K–Y model (Eq. (1)), are displayed for different values of Re and We . The CFD model predictions in Fig. 1(a) and (b) are based on contact angle values of 45° and 70° , respectively. Obviously, the K–Y model predictions displayed in Fig. 1(a) and (b) are the same since the model does not account for wetting effects (Eq. (1)). The K–Y equation significantly overpredicts the maximum spreading ratio for $\theta = 70^\circ$, but predictions are quite close to the numerical results for $\theta = 45^\circ$. This finding is physically reasonable since wetting effects are not considered in the K–Y equation, and hence, the model cannot account for the retarding effect on spreading of a large contact angle. To correct this deficiency in the K–Y equation, a contact angle correction of the form given in Eq. (2) is proposed:

$$\beta_{\text{KY, corr}} = \beta_{\text{KY}} \times (45/\theta)^n. \quad (2)$$

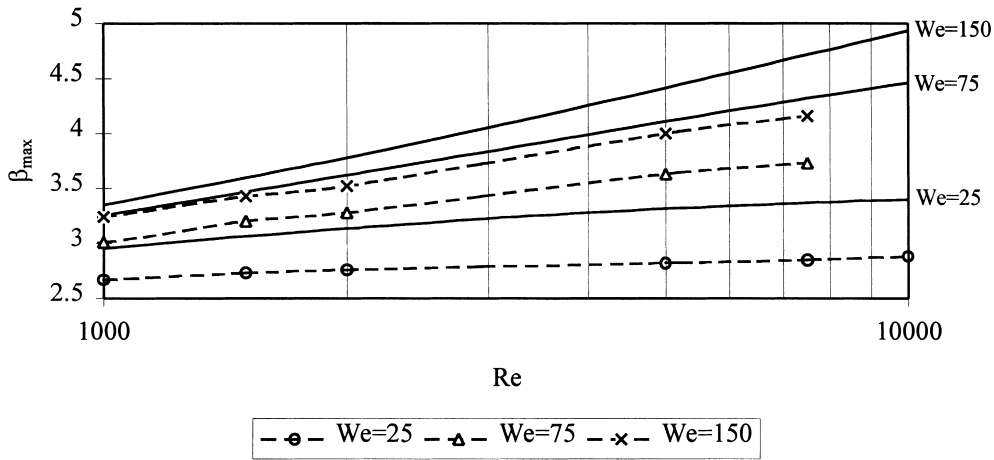
This correction will maintain the current predictions when the contact angle is near 45° , but it will adjust the predictions depending on the contact angle. Since a larger contact angle leads to a smaller β_{\max} , the exponent on the correction term will be positive. While this correction does not have a

physical basis, it maintains the energy balance characteristics of the original K–Y equation.

To determine the exponent in Eq. (2), a regression was performed to match the predictions of Eq. (2) to the results from the numerical model. The following parameter ranges were used in the numerical simu-

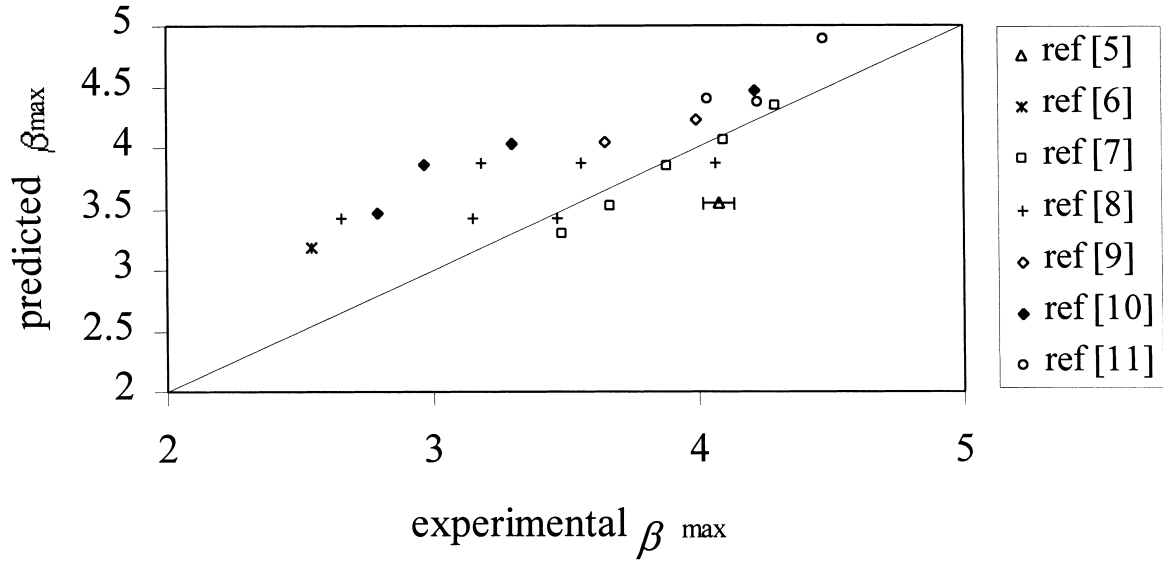


(a)

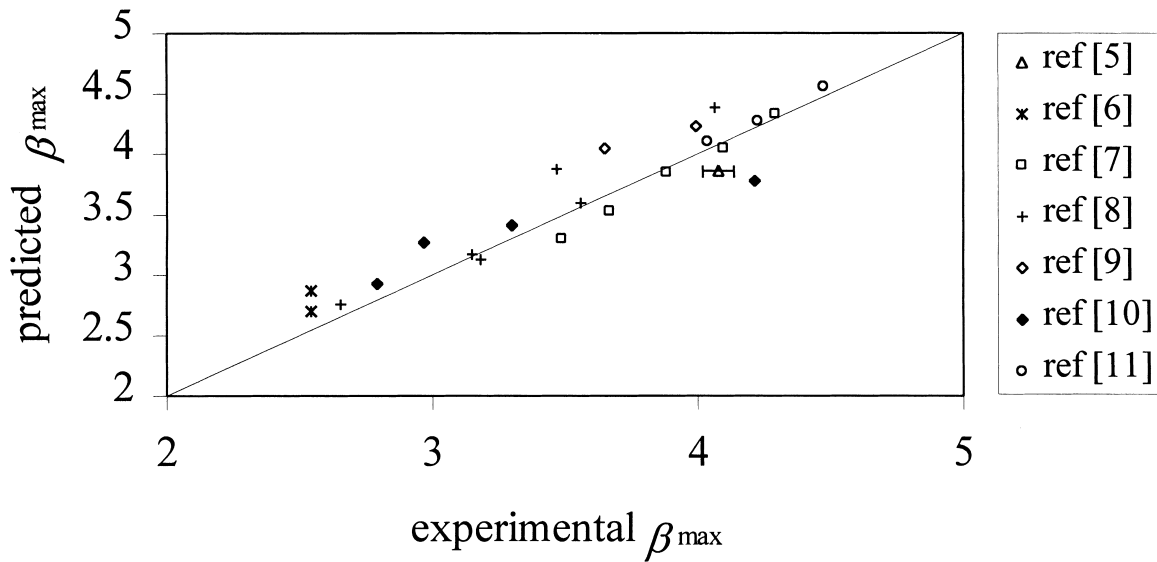


(b)

Fig. 1. Comparison between CFD model predictions [3] (discrete points) and the Kurabayashi–Yang predictions (solid lines). (a) $\theta = 45^\circ$ (b) $\theta = 70^\circ$.



(a)



(b)

Fig. 2. Comparison between experimental values of β_{\max} from indicated references and predicted values; (a) original equation (Eq. (1)), (b) modified equation (Eq. (3)). (Uncertainties for measurements from Refs. [6–11] were not reported.)

Table 1
Experimental data for β_{\max} comparisons

Reference	Liquid	Solid surface	Velocity range (m/s)	D (mm)	θ (degrees)
Chandra and Avedisian [5]	<i>n</i> -Heptane	Stainless steel	0.93	1.5	32
Tsurutani et al. [6]	Water	Unidentified metal	0.976	2.08	90 ^a
Valenzuela et al. [7]	Water	Glass	0.52–2.284	2	45 ^a
Ford and Furnidge [8]	Water	Glass, beeswax, cellulose acetate	2.607, 3.276	0.616	27, 62, 111
Toda [9]	Water	Glass	1.73	2.2–2.6	45 ^a
Shi and Chen [10]	Water	Aluminum	1.06–2.03	2.48–4.72	90 ^a
Fukai et al. [11]	Water	Treated glass	1.48–2.09	3.7	49, 60, 70, 92

^a Values estimated for the liquid/solid combination based on reported data [12–15].

lations to determine the maximum spreading ratio as a function of the input parameters:

$$25 \leq We \leq 150, \quad 1000 \leq Re \leq 7500, \quad 30^\circ \leq \theta \leq 90^\circ.$$

The resulting correction is

$$\beta_{KY, \text{corr}} = \beta_{KY} \times (45/\theta)^{0.241}. \quad (3)$$

Clearly, the above correction does not result in a “perfect” match between $\beta_{KY, \text{corr}}$ and the calculated values using the detailed CFD model. The ultimate test, however, is whether the above correction improves the model’s predictions vis-à-vis experimental data, as described below.

3. Comparison with experimental data

Predictions from the modified K–Y model for β_{\max} (Eq. (3)) were compared to experimental data obtained from the literature for the parameter ranges considered. Table 1 lists information on those data. Only data involving droplets impacting with $We < 150$ are used for comparison, and all data points used involve impact with a known contact angle determined at room temperature. That contact angle was either given in the original reference or determined for the particular combination of liquid and solid based on values from the literature [12–15], as indicated in Table 1. Uncertainties in the measurements were given only in reference [5]. Based on that discussion, the expanded uncertainty of the measurement of the spreading ratio reported in [5] is computed as ± 0.056 using a coverage factor of 2. Expanded uncertainties for measurements from the other sources are unknown.

Predictions for the maximum spreading ratio were calculated at each data point using the original K–Y equation (Eq. (1)), and the corrected K–Y (Eq. (3)). These predictions are plotted against the experimental values of β_{\max} in Fig. 2(a) and (b). In these plots, per-

fect predictions would fall along the 45° line. Predictions from the original K–Y equation tend to lie above the 45° line, indicating that the model tends to over-predict the maximum spreading ratio for the data studied. Predictions from the corrected equations, however, cluster around the 45° line, indicate that the predictions are much closer to the experimental values. To determine the improved accuracy of the modified equation, average percent errors of the predictions were computed along with the expanded uncertainty using a coverage factor of 2 to approximately yield a 95% confidence interval on the errors. The percent errors decreased from $12.2 \pm 24.8\%$ for the original correlation (Eq. (1)) to $3.6 \pm 12.2\%$ for the modified equation (Eq. (3)). As mentioned previously, an improved estimation of β_{\max} leads to substantially improved predictions of heat transfer because the wetted area is a function of the square of β_{\max} . The improvement shown by this correction, then, should provide a much more accurate prediction of the geometry involved in the heat transfer to the film formed by an impacting droplet.

4. Conclusion

Wetting effects have been incorporated into the K–Y equation for predicting the maximum spreading ratio of a liquid film formed by an impacting droplet by adding a correction based on the contact angle between the spreading liquid and the solid surface. A detailed numerical model [3,4] was used to estimate the correction. Comparison of the corrected equation to experimental data from the literature has shown significant improvement in predictions of β_{\max} . The improved model can be used to more accurately predict the area of the liquid film formed by the impacting droplet. Such predictions should greatly aid in modeling the heat transfer to this film in applications such as spray cooling.

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